satisfied by the *four* values $\theta=\lambda$, $\theta=\mu$, $\theta=\nu$, $\theta=\rho$, in virtue of the given equations; hence it must be an identity.

To find the value of x, multiply up by $a+\theta$, and then put $a+\theta=0$; thus

$$x=\frac{(a+\lambda)(a+\mu)(a+\nu)(a+\rho)}{(a-b)(a-c)(a-d)}.$$

By symmetry, we have

$$y=\frac{(b+\lambda)(b+\mu)(b+\nu)(b+\rho)}{(b-c)(b-d)(b-a)},$$

$$z=\frac{(c+\lambda)(c+\mu)(c+\nu)(c+\rho)}{(c-d)(c-a)(c-b)},$$

$$\text{and } u=\frac{(d+\lambda)(d+\mu)(d+\nu)(d+\rho)}{(d-a)(d-b)(d-c)}.$$

Similarly solved by *G. B. M. ZERR.*

GEOMETRY.

185. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Given the tangential equations to two conics S, S', find the tangential co-ordinates of the join of the poles of two given parallel lines with respect to S. Deduce the tangential equation of the center of S, and find that of the intersection of S and S'.

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $bc-f^2=A$, $ca-g^2=B$, $ab-h^2=C$, $gh-af=F$, $hf-bg=G$, $fg-ch=H$.

Then $S=A\lambda^2+B\mu^2+C\nu^2+2F\mu\nu+2G\nu\lambda+2H\lambda\mu$.

Similarly, $S'=A'\lambda^2+B'\mu^2+C'\nu^2+2F'\mu\nu+2G'\nu\lambda+2H'\lambda\nu$.

Let $\lambda\alpha+\mu\beta+\nu\gamma$ and $\lambda\alpha+\mu\beta+\nu\gamma+m$ be the two given parallel lines; p, q, t and p', q', t' their poles with respect to S. Then for the first line

$$ap+hq+gt=\lambda,\ hp+bq+ft=\mu,\ gp+fq+ct=\nu.$$

Solving these equations for p, q, t,

$$p=(A\lambda+H\mu+G\nu)/\triangle,\ q=(H\lambda+B\mu+F\nu)/\triangle,$$
$$t=(G\lambda+F\mu+C\nu)/\triangle, \text{ where } \triangle=abc+2fgh-af^2-bg^2-ch^2.$$

For the second line,

$$ap'+hq'+gt'+m=\lambda,\ hp'+bq'+ft'+m=\mu,\ gp'+fq'+ct'+m=\nu.$$

$\therefore p'=[A\lambda+H\mu+G\nu-m(A+H+G)]/\triangle.$

$q'=[H\lambda+B\mu+F\nu-m(H+B+F)]/\triangle.$

$t'=[G\lambda+F\mu+C\nu-m(G+F+C)]/\triangle.$

(p, q, t), (p', q', t') are the tangential co-ordinates of the join of the poles.

Let A', B', C' be the angles of the triangle of reference. The center is the pole of the line at infinity $\alpha\sin A'+\beta\sin B'+\gamma\sin C'=0$. The tangential co-ordinates of the center are obtained by substituting $\sin A'$, $\sin B'$, $\sin C'$ for λ, μ, ν in p, q, t and are

$$S_1=(A\sin A'+H\sin B'+G\sin C')/\triangle,$$

$$S_2=(H\sin A'+B\sin B'+F\sin C')/\triangle,$$

$$S_3=(G\sin A'+F\sin B'+C\sin C')/\triangle.$$

$\therefore$ The tangential equation of the center is $\lambda S_1+\mu S_2+\nu S_3=0$.

Write $a+ka'$ for a, $b+kb'$ for b, $c+kc'$ for c, $f+kf'$ for f, $g+kg'$ for g, $h+kh'$ for h in $A\lambda^2+B\mu^2+C\nu^2+2F\mu\nu+2G\nu\lambda+2H\lambda\mu=0$.

Then the tangential equation of the four points of intersection of S and S' is $S+k\Phi+k^2S'=0$ where k is undetermined, and

$$\Phi=(bc'+b'c-2ff')\lambda^2+(ca'+c'a-2gg')\mu^2+(ab'+a'b-2hh')\nu^2$$
$$+2(gh'+g'h-af'-a'f)\mu\nu+2(hf'+h'f-bg'-b'g)\pi\lambda$$
$$+2(fg'+f'g-ch'-c'h)\lambda\mu.$$

The condition for equal roots for k is $\Phi^2=4SS'$, which is the equation of the four points of intersection.

186. Proposed by J. R. HITT; Professor of Mathematics, Coronal Institute, San Marcos, Texas.

If two sides of a triangle and its in-circle be given in position, tne envelope of its circumcircle is a circle (*Mannheim*). [From Casey's *Sequel to Euclid.*]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let vertex A be origin, sides b, c the axes. Then $x^2+2xy\cos A+y^2-bx-cy=0$ is the equation to the circumcircle. Let this equation be written

$$D-bx-cy=0....(1).$$

Since the sides b, c and the inscribed circle are fixed in position, the tangents from A to the in-circle are constant.

$\therefore b+c-a=$ a constant$=m....(2)$.

$a=\sqrt{(b^2+c^2-2bc\cos A)}$. This in (2) gives after reduction,

$$m^2+2bc(1+\cos A)-2m(b+c)=0....(3).$$